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MADALGO seminars by Jeff Erickson, University of Illinois

Tracing Curves on Triangulated Surfaces.

Abstract:
Let σ be an arbitrary triangulated surface, possibly with boundary, composed of *n* triangles. A simple path or cycle in σ is normal if it avoids every vertex of σ and it does not cross any edge of σ twice in a row. We describe an algorithm to trace an arbitrary normal curve in $O(\min(X, n^2 \log x))$ time, where X is the number of times the curve crosses edges of the input triangulation. In particular, our algorithm runs in polynomial time even when the number of crossings is exponential in *n*. Our tracing algorithm computes a new cellular decomposition of the surface with complexity *O*(*n*); the traced curve appears as a simple path or cycle in the 1-skeleton of the new decomposition.

Our tracing strategy implies fast algorithms for (multi)curves represented by normal coordinates, which record the number of times the curves cross each edge of the surface triangulation. For example, we can count the components of a normal curve, determine whether two disjoint normal curves are isotopic, or determine whether a normal curve disconnects two given points on the surface, all in $O(\min(X, n^2 \log x))$ time. Our results are competitive with and conceptually simpler than earlier algorithms by Schaefer, Sedgwick, and Stefankovic [COCOON 2002] based on word equations and text compression.

As another application, we describe an algorithm to trace simple *geodesics* in piecewise-linear surfaces, represented by a set of Euclidean triangles with pairs of equal-length edges identified. Given a starting point and a direction, in the local coordinates of one of the constituent triangles, our algorithm locates the first point of self-intersection of the resulting geodesic path in $O(\min(X, n^2 \log x))$ time.

Joint work with PhD student Amir Nayyeri (unpublished)